1. Given the function $f(x) = \dfrac{4x-32}{x^2-6x-16}$, answer the questions below. Show all work and reasoning for full credit.

(a) (3 points) Evaluate $\lim_{x\to\infty} f(x)$.

+2 +1

$$\lim_{x\to\infty} \frac{4x-32}{x^2-6x-16} = \lim_{x\to\infty} \frac{4(\cancel{x-8})}{(\cancel{x-8})(x+2)} = \lim_{x\to+\infty} \frac{4}{x+2} = \underline{\underline{0}}$$

(a) 0

(b) (3 points) Evaluate $\lim_{x\to 8} f(x)$.

$$\lim_{x\to 8} \frac{4}{x+2} = \frac{4}{10} = \underline{\underline{\frac{2}{5}}}$$

+3

(b) $\frac{2}{5}$

2. Let $V(t)$ represent the velocity of a fast vehicle (in meters per second) after t seconds have passed.

(a) (2 points) $V(52) = 43$. Explain the meaning of this statement about the vehicle using a complete sentence. You must include units in your answer.

+1

At 52 seconds, the velocity of the vehicle is 43 m/sec.

+1

(b) (2 points) $V'(52) = -12$. Explain the meaning of this statement about the vehicle using a complete sentence. You must include units in your answer.

After 52 seconds, the velocity of the vehicle is reduced by approx! 12 $\frac{m/sec}{sec}$ thereafter.

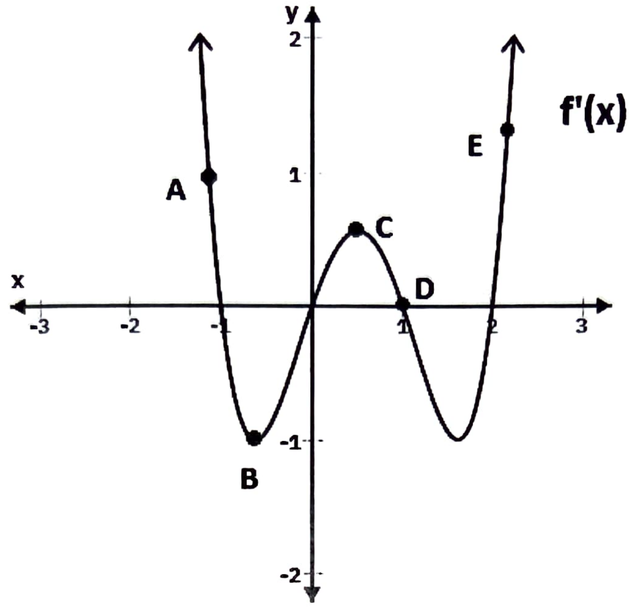
+1 +1

(c) (2 points) $\int_0^{52} V(t)\,dt = 5720$. Explain the meaning of this statement about the vehicle using a complete sentence. You must include units in your answer.

$\int_0^{52} V(t)\,dt = 5{,}720$ is the (net) distance in m from the origin after 52 seconds

+2

3. (7 points) Use the graph of $f'(x)$ below to answer questions about $f(x)$, $f'(x)$, and $f''(x)$. For each part, circle all of the correct responses. Choose NA if none of the responses apply. CAUTION: **The graph below is that of $f'(x)$ and not $f(x)$.**



(a) At which point(s), if any, is $f(x)$ decreasing?

A B C D E NA +1

(b) At which point(s), if any, is $f'(x)$ decreasing?

A B C D E NA +1

(c) At which point(s), if any, is $f''(x)$ approximately zero?

A B C D E NA +2

(d) Which point(s), if any, are critical points of $f(x)$?

A B C D E NA +1

(e) Which point(s), if any, are inflection points of $f(x)$?

A B C D E NA +1

(f) At which point(s), if any, are both $f'(x)$ and $f''(x)$ both negative?

A B C D E NA +1

4. Find the derivatives below. You are not required to simplify your final answer.

(a) (3 points) $f'(\theta)$ for $f(\theta) = 6\theta e^{\cos(\theta)}$

$$f'(\theta) = 6\Big(\overbrace{\theta e^{\cos(\theta)} \cdot -\sin(\theta)}^{+1} + \overbrace{e^{\cos(\theta)}}^{+1}\Big)$$

$$= \underbrace{-6\theta e^{\cos(\theta)}\sin(\theta) + 6e^{\cos(\theta)}}_{+1}$$

(b) (3 points) $h'(x)$ for $h(x) = \sqrt{2x^2 + x + \frac{2}{x^3}}$

$$h'(x) = \underbrace{\tfrac{1}{2}\left(2x^2 + x + \tfrac{2}{x^3}\right)^{-\frac{1}{2}}}_{+2} \underbrace{\left(4x + 1 - 6x^{-4}\right)}_{+1}$$

(c) (3 points) $w'(t)$ for $w(t) = \ln(t^2 - 1)$

$$w'(t) = \underbrace{\frac{1}{t^2-1}}_{+2} \underbrace{(2t)}_{+1} = \frac{2t}{t^2-1}$$

5. Suppose $g(x) = x^2 + x$.

(a) (1 point) Find the value of $g(3)$.

$$g(3) = 3^2 + 3 = 9 + 3 = \underline{\underline{12}} \quad +1$$

(b) (2 points) Expand and simplify completely: $g(3 + h)$.

$$g(3+h) = (3+h)^2 + (3+h) \quad +1$$
$$= 9 + 6h + h^2 + 3 + h$$
$$= \underline{\underline{h^2 + 7h + 12}} \quad +1$$

(c) (2 points) Write an expression for $g'(3)$ **using the limit definition** of the derivative.

$$g'(3) = \underbrace{\lim_{h \to 0}}_{+1} \underbrace{\frac{g(h+3) - g(3)}{h}}_{+1}$$

(d) (2 points) Evaluate $g'(3)$ using your expression in part (c).

$$= \lim_{h \to 0} \frac{h^2 + 7h + 12 - 12}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(h+7)}{\cancel{h}} \quad +1$$

$$= \lim_{h \to 0} h + 7$$

$$= \underline{\underline{7}} \quad +1$$

6. A public health agency is studying the spread of a new respiratory disease. They have developed a mathematical model to predict the percentage of the population that is infected over time. The percentage of the population infected, $P(t)$, is given by the function:

$$P(t) = \frac{6t}{t^2 + 49}$$

where t represents the number of weeks since the first case was confirmed. The model is considered valid for the first 14 weeks ($0 \leq t \leq 14$) of the outbreak.

(a) (3 points) Find $P(0)$. Include units and explain the contextual meaning of your answer in a single sentence.

+1

$$P(0) = \frac{6 \cdot 0}{0^2 + 49} = 0\ \% \text{ of population}$$

+1

There are no inflections at time $t = 0$. +1

(b) (6 points) Find any critical points of $P(t)$. You must use calculus to show that they are critical points. Show all work and reasoning to receive credit.

$$P'(t) = \frac{(t^2+49)(6) - 6t(2t)}{(t^2+49)^2} = \frac{6t^2 + 294 - 12t^2}{(t^2+49)^2} \quad +2$$

$$= \frac{-6t^2 + 294}{(t^2+49)^2} = 0 \Rightarrow 6t^2 = 294 \Rightarrow t^2 = 49$$

+2 $\Rightarrow t = 7, -7$

extraneous

$t = 7$ +2

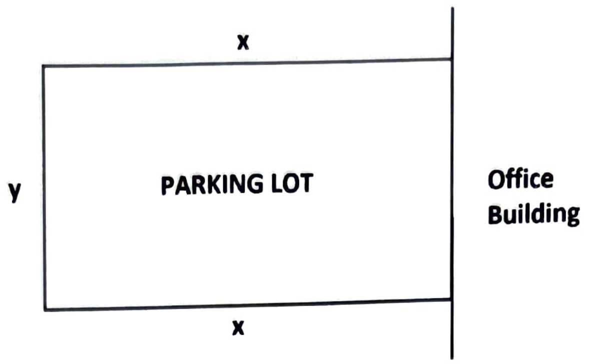
(c) (2 points) What is the maximum proportion of the population that will be infected during this 14-week period, and when will this maximum occur? You must use calculus to receive credit for your answer.

The maximum proportion occurs at 7 weeks. +1

$$P(7) = \frac{42}{2(49)} = \frac{42}{98} = 0.4286$$

So, 42.86% of the population is infected at 7 weeks. +1

7. A building contractor wants to build a rectangular fenced parking lot next to an office building. The fencing needs to enclose an area of 9000 square feet for 60 parking spaces. The east side of the parking lot will be adjacent to the building, so the lot only requires three sides of fencing.



(a) (2 points) Write an equation involving x and y for the total length of new fencing (in feet) the contractor needs to build the parking lot.

$$L(x,y) = \underbrace{2x}_{+1} + \underbrace{y}_{+1}$$

(b) (3 points) Write an equation for the total length needed involving only x.

$$\underbrace{xy = 9000}_{+1} \Rightarrow \underbrace{y = \frac{9000}{x}}_{+1}$$

$$\therefore \; L(x) = \underbrace{2x + \frac{9000}{x}}_{+1}$$

(c) (5 points) What is the minimum length of fencing that needs to be purchased to build the parking lot? Round your answer to one decimal place.

$$L'(x) = \underbrace{2 - \frac{9000}{x^2}}_{+2} = 0 \Rightarrow 2x^2 = 9000 \Rightarrow x = \pm\sqrt{4500}$$

$$\Rightarrow x = \underbrace{67.082}_{+2}, \underbrace{-67.082}_{\text{extraneous}}$$

$$\therefore \; L(x) = 2(67.082) + \frac{9000}{67.082} = \underbrace{268.3 \text{ ft}}_{+1}$$

8. (4 points) A company produces q electronic devices every week. The company calculates its costs using the function $C(q)$ and its revenue using the function $R(q)$.

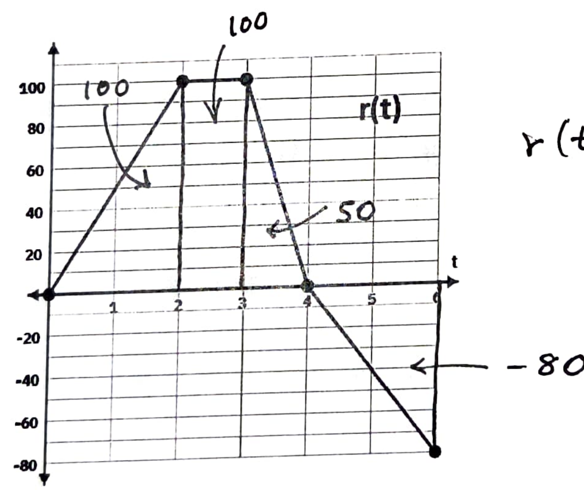
If $C'(250) = 49$ and $R'(250) = 54$, should the quantity produced be increased or decreased from $q = 250$ electronic devices to increase profits? You must explain your answer in a single sentence.

+2

Since $R'(250) = 54 > C'(250) = 49$, production should be increased to increase profit.

+2

9. (5 points) The function $r(t)$ is the rate of change (or derivative) of a function $R(t)$. Using the figure of $r(t)$ below, find $R(6)$ if $R(0) = 10$.



$r(t) = R'(t)$

+2

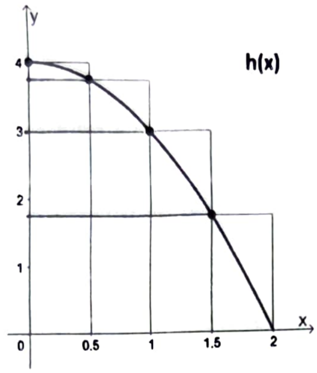
$$170 = 250 - 80 = \int_0^6 r(t)\,dt = R(6) - R(0)$$

+2

$$\Rightarrow R(6) = 170 + 10 = 180$$

$R(6) =$ 180 +1

10. The figure shows the graph of a function $h(x)$ for $0 \le x \le 2$ along with the rectangles used to approximate the definite integral $\int_0^2 h(x)\,dx$



(a) (1 point) Do the rectangles represent a left or right Riemann sum?

Left sum +1

(b) (1 point) What is the value of n, the number of subdivisions used?

$n = 4$ +1

(c) (1 point) What is the value of Δx?

$\Delta x = \frac{1}{2}$ +1

(d) (2 points) Use the figure above to estimate the value of $\int_0^2 h(x)\,dx$ with Riemann sums.

$$\int_0^2 f(x)\,dx \cong \frac{1}{2}\left(\frac{7}{4} + 3 + \frac{15}{4} + 4\right) \quad +1$$

$$= \frac{1}{2}\left(\frac{7}{4} + \frac{15}{4} + \frac{12}{4} + \frac{16}{4}\right) = \frac{1}{2}(50) = \underline{\underline{25}} \quad +1$$

11. Given that $f(x)$ is an ~~[illegible]~~ continuous function and that $\int_0^5 f(x)\,dx = 12$, answer the questions below.

(a) (4 points) Find $\int_0^5 (3f(x) - 2)\,dx$

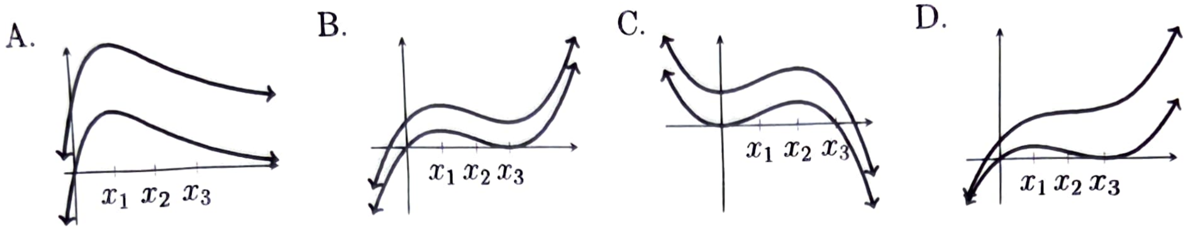
$$\int_0^5 (3f(x) - 2)\,dx = \underbrace{3\int_0^5 f(x)\,dx - \int_0^5 2\,dx}_{+2} = 3(12) - 2(5)$$

$$= \underline{\underline{26}} \quad +2$$

(b) (2 points) Find the average value of $f(x)$ over the interval [0,5].

$$\text{Ave. value} = \underbrace{\frac{1}{5-0}\int_0^5 f(x)\,dx}_{+1} = \frac{1}{5}(12) = \underline{\underline{\frac{12}{5}}} \quad +1$$

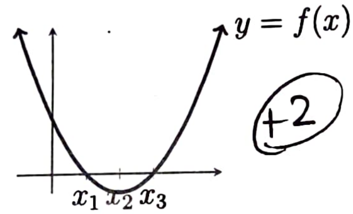
12. Consider the pairs of functions graphed below.



(a) (2 points) Which of the pairs represent possible graphs of antiderivatives of a function g with $g(x_1) = 0$? Mark all correct choices below.

● A ● B ○ C ○ D +2

(b) (2 points) Below is a graph of $f(x)$. Which of the pairs represent possible graphs of F, antiderivatives of the function f shown in the following graph? Mark all correct choices below.



+2

○ A ● B ○ C ○ D

13. Find the exact values of the definite integrals below. You must use the Fundamental Theorem of Calculus. Answers without relevant calculus work will receive no credit.

(a) (4 points) $\int_1^2 \left(\frac{10}{x^2} - 4x\right) dx$ +2

$$= \frac{-10}{x} - 2x^2 \Big|_1^2$$

$$= -\frac{10}{2} - 8 - (-10 - 2) = -5 - 8 + 12 = \underline{\underline{-1}}$$ +2

(b) (4 points) $\int_0^\pi (4\sin(t) - 5\cos(t) + 1)\, dt$ +2

$$-4\cos(t) - 5\sin(t) + t \Big|_0^\pi$$

$$= -4\cos(\pi) - 5\underbrace{\sin(\pi)}_{=0} + \pi - (-4\cos(0) - 5\sin(0))$$

$$= 4 + \pi + 4 = \underline{\underline{8 + \pi}}$$ +2